

CBSE Class 11 Mathematics
Important Questions
Chapter 2
Relations and Functions

1 Marks Questions

1. Find a and b if $(a - 1, b + 5) = (2, 3)$ If $A = \{1,3,5\}$, $B = \{2,3\}$ find : (Question-2, 3)

Ans. $a = 3, b = -2$

2. $A \times B$

Ans. $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

3. $B \times A$ Let $A = \{1,2\}$, $B = \{2,3,4\}$, $C = \{4,5\}$, find (Question- 4,5)

Ans. $B \times A = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$

4. $A \times (B \cap C)$

Ans. $\{(1,4), (2,4)\}$

5. $A \times (B \cup C)$

Ans. $\{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)\}$

6. If $P = \{1,3\}$, $Q = \{2,3,5\}$, find the number of relations from A to B

Ans. $2^6 = 64$

7. If $A = \{1,2,3,5\}$ and $B = \{4,6,9\}$, $R = \{(x, y) : |x - y| \text{ is odd, } x \in A, y \in B\}$ Write R in roster form

Which of the following relations are functions. Give reason.

Ans. $R = \{ (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6) \}$

8. $R = \{ (1,1), (2,2), (3,3), (4,4), (4,5) \}$

Ans. Not a function because 4 has two images.

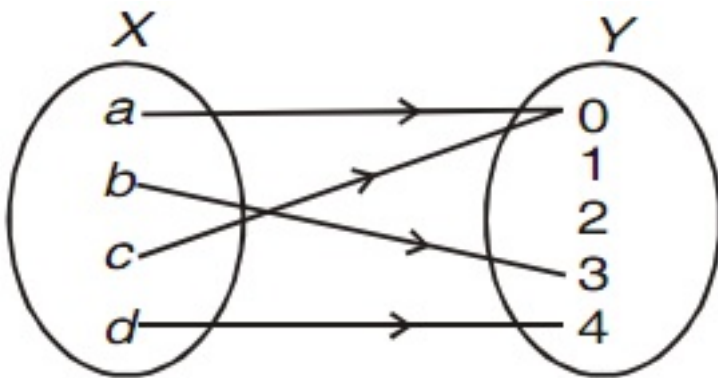
9. $R = \{ (2,1), (2,2), (2,3), (2,4) \}$

Ans. Not a function because 2 does not have a unique image.

10. $R = \{ (1,2), (2,5), (3,8), (4,10), (5,12), (6,12) \}$ Which of the following arrow diagrams represent a function? Why?

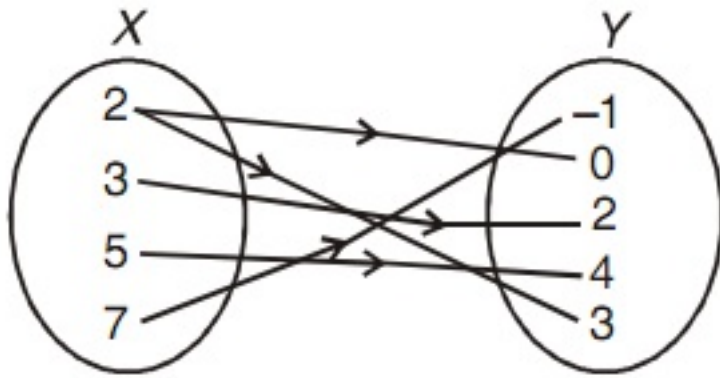
Ans. Function

11.



Ans. Function

12.



Let f and g be two real valued functions, defined by, $f(x) = x^2$, $g(x) = 3x + 2$.

Ans. Not a function

13. $(f + g)(-2)$

Ans. 0

14. $(f - g)(1)$

Ans. -4

15. $(fg)(-1)$

Ans. -1

16. $\left(\frac{f}{g}\right)(0)$

Ans. 0

17. If $f(x) = x^3$, find the value of, $\frac{f(5) - f(1)}{5 - 1}$

Ans. 31

18. Find the domain of the real function, $f(x) = \sqrt{x^2 - 4}$

Ans. $(-\infty, -2] \cup [2, \infty)$

19. Find the domain of the function, $f(x) = \frac{x^2 + 2x + 3}{x^2 - 5x + 6}$ Find the range of the following functions, (Question- 20,21)

Ans. $\mathbb{R} - \{2,3\}$

20. $f(x) = \frac{1}{1-x^2}$

Ans. $(-\infty, -0] \cup [1, \infty)$

21. $f(x) = x^2 + 2$

Ans. $[2, \infty)$

22. Find the domain of the relation, $R = \{(x, y) : x, y \in \mathbb{Z}, xy = 4\}$ Find the range of the following relations : (Question-23, 24)

Ans. $\{-4, -2, -1, 1, 2, 4\}$

23. $R = \{(a,b) : a, b \in \mathbb{N} \text{ and } 2a + b = 10\}$

Ans. $\{2,4,6,8\}$

24. $R = \left\{ \left(x, \frac{1}{x} \right) : x \in \mathbb{Z}, 0 < x < 6 \right\}$

Ans. $\left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \right\}$

25. If the ordered Pairs $(x-1, y+3)$ and $(2, x+4)$ are equal, find x and y

(i) (3,3) (ii) (3,4) (iii) (1,4) (iv) (1,0)

Ans. (3,4)

26. If, $n(A) = 3, n(B) = 2$, A And B are two sets Then no. of relations of $A \times B$ have.

(i) (6) (ii) (12) (iii) (32) (iv) (64)

Ans. 64

27. Let $f(x) = -|x|$ then Range of function

(i) $(0, \infty)$ (ii) $(-\infty, \infty)$ (iii) $(-\infty, 0)$ (iv) none of there

Ans. $(-\infty, 0)$

28. A real function f is defined by $f(x) = 2x - 5$. Then the Value of $f(-3)$

(i) -11 (ii) 1 (iii) 0 (iv) none of there

Ans. -11

29. If $P = \{a, b, c\}$ and $Q = \{d\}$, form the sets $P \times Q$ and $Q \times P$ are these two Cartesian products equal?

Ans. Given $P = \{a, b, c\}$ and $Q = \{d\}$, by definition of cartesian product, we set

$$P \times Q = [(a, d), (b, d), (c, d)] \text{ and } Q \times P = [(d, a), (d, b), (d, c)]$$

By definition of equality of ordered pairs the pair (a, d) is not equal to the pair (d, a) therefore $P \times Q \neq Q \times P$.

30. If A and B are finite sets such that $n(A) = m$ and $n(B) = k$ find the number of relations from A to B

Ans. Given $n(A) = m$ and $n(B) = k$

$$\therefore n(A \times B) = \cap(A) \times \cap(B) = mk$$

\therefore the number of subsets of $A \times B = 2^{mk}$

$\because n(A) = m$, then the number of subsets of $A = 2^m$

Since every subset of $A \times B$ is a relation from A to B therefore the number of relations from A to $B = 2^{mk}$

31. Let $f = \{(1,1), (2,3), (0,-1), (-1,3), \dots\}$ be a function from \mathbb{Z} to \mathbb{Z} defined by $f(x) = ax + b$, for some integers a and b determine a and b .

Ans. Given $f(x) = ax + b$

Since $(1,1) \in f, f(1) = 1 \Rightarrow a + b = 1 \dots (i)$

$(2,3) \in f, f(2) = 3 \Rightarrow 2a + b = 3 \dots (ii)$

Subtracting (i) from (ii) we get $a = 2$

Substituting $a = 2$ in (i) we get $2 + b = 1$

$$\Rightarrow b = -1$$

Hence $a = 2, b = -1$

32. Express $\{(x, y) : y + 2x = 5, x, y \in \mathbb{W}\}$ as the set of ordered pairs

Ans. Since $y + 2x = 5$ and $x, y \in \mathbb{W}$,

$$\text{Put } x = 0, y + 0 = 5 \Rightarrow y = 5$$

$$x=1, y+2 \times 1=5 \Rightarrow y=3$$

$$x=2, y+2 \times 2=5 \Rightarrow y=1$$

For another values of $x \in W$, we do not get $y \in W$.

Hence the required set of ordered pairs is $\{(0, 5), (1, 3), (2, 1)\}$

33. If $A = \{1, 2\}$, find $(A \times A \times A)$

Ans. We have

$$A \times A \times A = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (2, 2, 1), (2, 2, 2)\}$$

34. A function f is defined by $f(x) = 2x - 3$ find $f(5)$

Ans. Here $f(x) = 2x - 3$

$$f(x) = (2 \times 5 - 3) = 7$$

35. Let $f = \{(0, -5), (1, -2), (2, 1), (3, 4), (4, 7)\}$ be a linear function from Z into Z find f

Ans. $f(x) = 3x - 5$

36. If the ordered pairs $(x - 2, 2y + 1)$ and $(y - 1, x + 2)$ are equal, find x & y

Ans. $x = 3, \quad y = 2$

37. Let $A = \{-1, 2, 5, 8\}$, $B = \{0, 1, 3, 6, 7\}$ and R be the relation, is one less than from A to B then find domain and Range of R

Ans. Given $A = \{-1, 2, 5, 8\}$, $B = \{0, 1, 3, 6, 7\}$, and R is the relation 'is one less than' from

A to B therefore $R = [(-1,0), (2,3), (5,6)]$

Domain of $R = \{-1, 2, 5\}$ and range of $R = \{0, 3, 6\}$

38. Let R be a relation from N to N define by $R = [(a, b) : a, b \in N \text{ and } a = b^2]$.

Is the following true $a, b \in R$ implies $(b, a) \in R$

Ans. No; let $a = 4, b = 2$. As $4 = 2^2$, so $(4, 2) \in R$ but $2 \neq 4^2$, so $(2, 4) \notin R$

39. Let N be the set of natural numbers and the relation R be define in N by $R = [(x, y) : y = 2x, x, y \in N]$. what is the domain, co domain and range of R ? Is this relation a function?

Ans. Given $R = [(x, y) : y = 2x, x, y \in N]$

\therefore Domain of $R = N$, co domain of $R = N$, and Range of R is the set of even natural numbers.

Since every natural number x has a unique image $2x$ therefore, the relation R is a function.

40. Let $R = \{(x, y) : y = x + 1\}$ and $y \in \{0, 1, 2, 3, 4, 5\}$ list the element of R

Ans. $R = \{(-1, 0), (0, 1), (1, 2), (2, 3), (3, 4), (4, 5)\}$

41. Let f be the subset of $Q \times Z$ defined by

$f = \left\{ \left(\frac{m}{n}, m \right) : m, n \in Z, n \neq 0 \right\}$. Is f a function from Q to Z ? Justify your answer

Ans. f Is not a function from Q to Z

$$f\left(\frac{1}{2}\right) = 1 \text{ and } f\left(\frac{2}{4}\right) = 2$$

$$\text{But } \frac{1}{2} = \frac{2}{4}$$

∴ One element $\frac{1}{2}$ have two images

∴ f is not function

42. The function ' f ' which maps temperature in Celsius into temperature in Fahrenheit is defined by $f(c) = \frac{9}{5}c + 32$ find $f(0)$

$$\text{Ans. } f(0) = \frac{9}{5} \times 0 + 32$$

$$f(0) = 32$$

43. If $f\left(x = x^3 - \frac{1}{x^3}\right)$ Prove that $f(x) + f\left(\frac{1}{x}\right) = 0$

$$\text{Ans. } f(x) = x^3 - \frac{1}{x^3}$$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$$

$$f(x) + f\left(\frac{1}{x}\right) = \cancel{x^3} - \cancel{\frac{1}{x^3}} + \cancel{\frac{1}{x^3}} - \cancel{x^3} = 0$$

44. If A and B are two sets containing m and n elements respectively how many different relations can be defined from A to B ?

$$\text{Ans. } 2^{m \times n}$$

CBSE Class 12 Mathematics
Important Questions
Chapter 2
Relations and Functions

4 Marks Questions

1. Let $A = \{1,2,3,4\}$, $B = \{1,4,9,16,25\}$ and R be a relation defined from A to B as, $R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2\}$

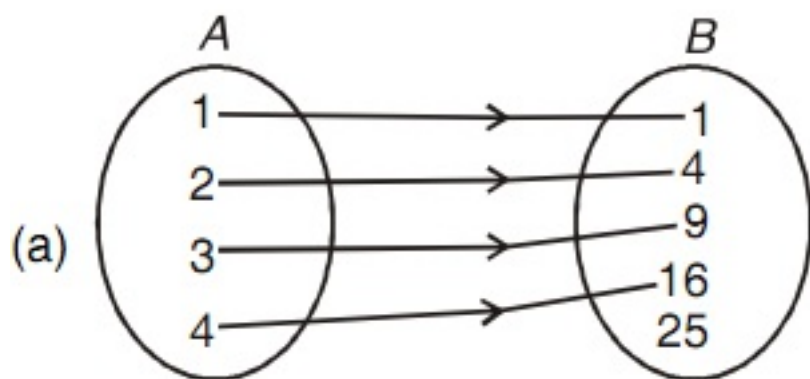
(a) Depict this relation using arrow diagram.

(b) Find domain of R .

(c) Find range of R .

(d) Write co-domain of R .

Ans.



(b) $\{1,2,3,4\}$

(c) $\{1,4,9,16\}$

(d) $\{1,4,9,16,25\}$

2. Let $R = \{(x, y) : x, y \in \mathbb{N} \text{ and } y = 2x\}$ be a relation on \mathbb{N} . Find :



(i) Domain

(ii) Codomain

(iii) Range

Is this relation a function from N to N

Ans. (i) N

(ii) N

(iii) Set of even natural numbers

yes, R is a function from N to N.

3. Find the domain and range of, $f(x) = |2x - 3| - 3$

Ans. Domain is R

Range is $[-3, \infty)$

4. Draw the graph of the Constant function, $f : R \rightarrow R; f(x) = 2 \quad x \in R$. Also find its domain and range.

Ans. Domain = R

Range = {2}

5. Let $R = \{(x, -y) : x, y \in W, 2x + y = 8\}$ then

(i) Find the domain and the range of R (ii) Write R as a set of ordered pairs.

Ans. (i) Given $2x + y = 8$ and $x, y \in W$

Put

$$x = 0, 2 \times 0 + y = 8 \Rightarrow y = 8,$$

$$x = 1, 2 \times 1 + y = 8 \Rightarrow y = 6,$$

$$x = 2, 2 \times 2 + y = 8 \Rightarrow y = 4,$$

$$x = 3, 2 \times 3 + y = 8 \Rightarrow y = 2,$$

$$x = 4, 2 \times 4 + y = 8 \Rightarrow y = 0$$

for all other values of $x \in W$, we do not get $y \in W$

\therefore Domain of $R = \{0, 1, 2, 3, 4\}$ and range of $R = \{8, 6, 4, 2, 0\}$

(ii) R as a set of ordered pairs can be written as

$$R = \{(0, 8), (1, 6), (2, 4), (3, 2), (4, 0)\}$$

6. Let R be a relation from Q to Q defined by $R = \{(a, b) : a, b \in Q \text{ and } a - b \in z\}$ show that (i) $(a, a) \in R$ for all $a \in Q$ (ii) $(a, b) \in R$ implies that $(b, a) \in R$

(iii) $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$

$$\text{Ans. } R = \{(a, b) : a, b \in Q \text{ and } a - b \in z\}$$

(i) For all $a \in Q$, $a - a = 0$ and $0 \in z$, it implies that $(a, a) \in R$.

(ii) Given $(a, b) \in R \Rightarrow a - b \in z \Rightarrow -(a - b) \in z$

$$\Rightarrow b - a \in z \Rightarrow (b, a) \in R.$$

(iii) Given $(a, b) \in R$ and $(b, c) \in R \Rightarrow a - b \in z$ and $b - c \in z \Rightarrow (a - b) + (b - c) \in z$

$$\Rightarrow a - c \in z \Rightarrow (a, c) \in R.$$

7. If $f(x) = \frac{x^2 - 3x + 1}{x - 1}$, find $f(-2) + f\left(\frac{1}{3}\right)$

Ans. Given $f(x) = \frac{x^2 - 3x + 1}{x - 1}$, $Df = R - \{1\}$

$$\therefore f(-2) = \frac{(-2)^2 - 3(-2) + 1}{-2 - 1} - \frac{4 + 6 + 1}{-3} = 1\frac{1}{3} \text{ and}$$

$$f\left(\frac{1}{3}\right) = \frac{\left(\frac{1}{3}\right)^2 - 3 \times \frac{1}{3} + 1}{\frac{1}{3} - 1} = \frac{\frac{1}{9} - 1 + 1}{-\frac{2}{3}} = \frac{\frac{1}{9}}{-\frac{2}{3}} = \frac{1}{9} \times \left(-\frac{3}{2}\right) = -\frac{1}{6}$$

$$\therefore f(-2) + f\left(\frac{1}{3}\right) = -\frac{11}{3} - \frac{1}{6} = \frac{-22 - 1}{6} = \frac{-23}{6} = 3\frac{5}{6}$$

8. Find the domain and the range of the function $f(x) = 3x^2 - 5$. Also find $f(-3)$ and the numbers which are associated with the number 43 in its range.

Ans. Given $f(x) = 3x^2 - 5$

For Df , $f(x)$ must be real number

$\Rightarrow 3x^2 - 5$ must be a real number

Which is a real number for every $x \in R$

$\Rightarrow Df = R \dots \dots \dots (i)$

for Rf , let $y = f(x) = 3x^2 - 5$

We know that for all $x \in R$, $x^2 \geq 0 \Rightarrow 3x^2 \geq 0$

$\Rightarrow 3x^2 - 5 \geq -5 \Rightarrow y \geq -5 \Rightarrow Rf = [-5, \infty]$

Further, as $-3 \in Df$, $f(-3)$ exists and $f(-3)$

$$= 3(-3)^2 - 5 = 22.$$

As $43 \in Rf$ on putting $y = 43$ in (i) we get

$$3x^2 - 5 = 43 \Rightarrow 3x^2 = 48 \Rightarrow x^2 = 16 \Rightarrow x = -4, 4.$$

Therefore -4 and 4 are number

(is Df) which are associated with the number 43 in Rf

9. If $f(x) = x^2 - 3x + 1$, find x such that $f(2x) = f(x)$

Ans. Given $f(x) = x^2 - 3x + 1, Df = R$

$$\therefore f(2x) = (2x)^2 - 3(2x) + 1 = 4x^2 - 6x + 1$$

As $f(2x) = f(x)$ (Given)

$$\Rightarrow 4x^2 - 6x + 1 = x^2 - 3x + 1$$

$$\Rightarrow 3x^2 - 3x = 0 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$$

$$\Rightarrow x = 0, 1.$$

10. Find the domain and the range of the function $f(x) = \sqrt{x-1}$

Ans. Given $f(x) = \sqrt{x-1}$,

for Df , $f(x)$ must be a real number

$$\Rightarrow \sqrt{x-1} \text{ must be a real number}$$

$$\Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$$

$$\Rightarrow Df = [1, \infty]$$

for Rf , let $y = f(x) = \sqrt{x-1}$

$$\Rightarrow \sqrt{x-1} \geq 0 \Rightarrow y \geq 0$$

$$\Rightarrow Rf = [0, \infty]$$

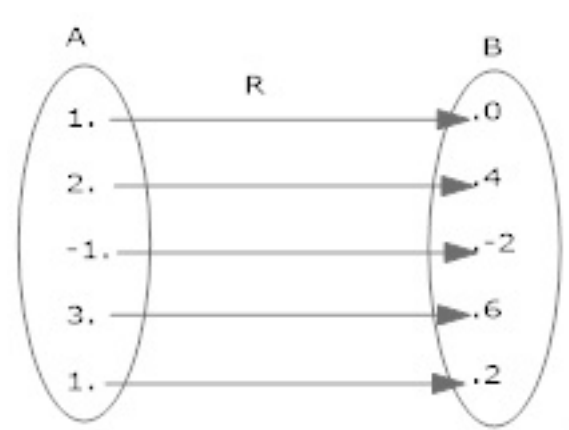
11. Let a relation $R = \{(0, 0), (2, 4), (-1, 2), (3, 6), (1, 2)\}$ then

(i) write domain of R

(ii) write range of R

(iii) write R the set builder form

(iv) represent R by an arrow diagram



Ans. Given $R = [(0, 0), (2, 4), (-1, -2), (3, 6), (1, 2)]$

(i) Domain of $R = [0, 2, -1, 3, 1]$

(ii) Rang of $R = [0, 4, -2, 6, 2]$

(iii) R in the builder form can be written as

$$R = [(x, y) : x \in I, -1 \leq x \leq 3, y = 2x]$$

(iv) The reaction R can be represented by the arrow diagram are shown.

12. Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$ and $R = \{(x, y) : (x, y) \in A \times B, y = x + 1\}$

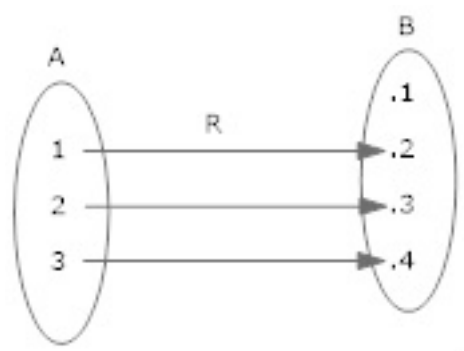
(i) find $A \times B$

(ii) write R in roster form

(iii) write domain & range of R

(iv) represent R by an arrow diagram

Ans. (i) $\{(1,1), (1,2), (1,3), (1,4)\}$



$(2,1), (2,2), (2,3), (2,4)$

$(3,1), (3,2), (3,3), (3,4)\}$

(ii) $R = [(1,2), (2,3), (3,4)]$

(iii) Domain of $R = \{1, 2, 3\}$ and range of $R = \{2, 3, 4\}$

(iv) The relation R can be represented by the arrow diagram as shown.

13. The cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set and the remaining elements of $A \times A$

Ans. Let $n(A) = m$

Given $n(A \times A) = 9 \Rightarrow n(A)n(A) = 9$

$\Rightarrow m.m = 9 \Rightarrow m^2 = 9 \Rightarrow m = 3 \quad (\because m > 0)$

Given $(-1, 0) \in A \times A \Rightarrow -1 \in A$ and $0 \in A$

Also $(0, 1) \in A \times A \Rightarrow 0 \in A$ and $1 \in A$

This $-1, 0, 1 \in A$ but $n(A) = 3$

Therefore $A = [-1, 0, 1]$

The remaining elements of $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$

14. Find the domain and the range of the following functions $f(x) = \frac{1}{\sqrt{5-x}}$

Ans. Given $f(x) = \frac{1}{\sqrt{5-x}}$

For D_f , $f(x)$ must be a real number

$\Rightarrow \frac{1}{\sqrt{5-x}}$ Must be a real number

$\Rightarrow 5-x > 0 \Rightarrow 5 > x \Rightarrow x < 5$

$\Rightarrow D_f = (-\infty, 5)$

For R_f let $y = \frac{1}{\sqrt{5-x}}$

As $x < 5, 0 < 5-x$

$\Rightarrow 5-x > 0 \Rightarrow \sqrt{5-x} > 0$

$\Rightarrow \frac{1}{\sqrt{5-x}} > 0 \left(\because \frac{1}{a} > 0 \text{ if and only if } a > 0 \right)$

$\Rightarrow y > 0$

$\Rightarrow R_f = (0, \infty)$

15. Let $f(x) = x+1$ and $g(x) = 2x-3$ be two real functions. Find the following

functions (i) $f+g$ (ii) $f-g$ (iii) fg (iv) $\frac{f}{g}$ (v) f^2-3g

Ans. Given $f(x) = x+1$ and $g(x) = 2x-3$ we note that $D_f = \mathbb{R}$ and $D_g = \mathbb{R}$ so these functions have the same Domain \mathbb{R}

(i) $(f+g)(x) = f(x) + g(x) = (x+1) + (2x-3) = 3x-2$, for $x \in \mathbb{R}$

(ii) $(f-g)(x) = f(x) - g(x) = (x+1) - (2x-3) = -x+4$, for all $x \in \mathbb{R}$

(iii) $(fg)(x) = f(x)g(x) = (x+1)(2x-3) = 2x^2 - x - 3$, for all $x \in \mathbb{R}$

(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}$, $x \neq \frac{3}{2}$, $x \in \mathbb{R}$

(v) $(f^2 - 3g)(x) = (f^2)(x) - (3g)(x) = (f(x))^2 - 3g(x)$

$$= (x+1)^2 - 3(2x-3) = x^2 + 2x + 1 - 6x + 9$$

$$= x^2 - 4x + 10, \text{ for all } x \in \mathbb{R}$$

16. Find the domain and the range of the following functions

(i) $f(x) = \frac{x-3}{2x+1}$ (ii) $f(x) = \frac{x^2}{1+x^2}$ (iii) $f(x) = \frac{1}{1-x^2}$

Ans. (i) Given $f(x) = \frac{x-3}{2x+1}$

For D_f , $f(x)$ must be a real number

$$\Rightarrow \frac{x-3}{2x+1} \text{ must be a real number}$$

$$\Rightarrow 2x+1 \neq 0 \Rightarrow x \neq -\frac{1}{2}$$

$$\Rightarrow D_f = \text{set of all real number except } -\frac{1}{2}$$

$$-\frac{1}{2} \text{ i.e. } R - \left[-\frac{1}{2} \right]$$

For R_F , let $y = \frac{x-3}{2x+1} \Rightarrow 2xy + y = x-3$

$$\Rightarrow (2y-1)x = -y-3 \Rightarrow x = \frac{y+3}{1-2y} \text{ but } x \in R$$

$$\Rightarrow \frac{y+3}{1-2y} \text{ Must be a real number } \Rightarrow 1-2y \neq 0 \Rightarrow y \neq \frac{1}{2}$$

$$\Rightarrow R_F = \text{Set of all real number except } \frac{1}{2} R - \left[\frac{1}{2} \right]$$

(ii) Given $f(x) = \frac{x^2}{1+x^2}$

For D_F , $f(x)$ must be a real number $\Rightarrow \frac{x^2}{1+x^2}$

Must be a real number

$$\Rightarrow D_F = R \quad (\because x^2 + 1 \neq 0 \text{ for all } x \in R)$$

For R_F let $y = \frac{x^2}{1+x^2} \Rightarrow x^2y + y = x^2$

$$\Rightarrow (y-1)x^2 = -y \Rightarrow x^2 = \frac{-y}{y-1}, y \neq 1$$

But $x^2 \geq 0$ for all $x \in R \Rightarrow \frac{-y}{y-1} \geq 0, y \neq 1$

Multiply both sides by $(y-1)^2$, a positive real number

$$\Rightarrow -y(y-1) \geq 0$$

$$\Rightarrow y(y-1) \leq 0 \Rightarrow (y-0)(y-1) \leq 0$$

$$\Rightarrow 0 \leq y \leq 1 \text{ but } y \neq 1$$

$$\Rightarrow 0 \leq y < 1$$

$$\Rightarrow R_f = (0,1)$$

(iii) Given $f(x) = \frac{1}{1-x^2}$

For $D_f, f(x)$ must be a real number

$$\Rightarrow \frac{1}{1-x^2} \text{ Must be a real number}$$

$$\Rightarrow 1-x^2 \neq 0 \Rightarrow x \neq -1,1$$

$$\Rightarrow D_f = \text{Set of all real number except } -1,1 \therefore D_f = R - [-1,1]$$

For R_f let $y = \frac{1}{1-x^2}, y \neq 0$

$$\Rightarrow 1-x^2 = \frac{1}{y} \Rightarrow x^2 = 1 - \frac{1}{y} \neq 0$$

But $x^2 \geq 0$ for all $x \in D_f \Rightarrow 1 - \frac{1}{y} \geq 0$

But $y^2 > 0, y \neq 0$

Multiplicity both sides by y^2 a positive real number

$$\Rightarrow y^2 \left(1 - \frac{1}{y}\right) \geq 0 \Rightarrow y(y-1) \geq 0 \Rightarrow (y-0)(y-1) \geq 0$$

Either $y \leq 0$ or $y \geq 1$ but $y \neq 0$

$$\Rightarrow R_F = (-\infty, 0) \cup (1, \infty).$$

17. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$

find (i) $A \times (B \cup C)$ (ii) $A \times (B \cap C)$ (iii) $(A \times B) \cap (B \times C)$

Ans. We have

$$(i) (B \cup C) = \{3, 4\} \cup \{4, 5, 6\} = \{3, 4, 5, 6\}$$

$$\therefore A \times (B \cup C)$$

$$= \{1, 2, 3\} \times \{3, 4, 5, 6\}$$

$$= \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4),$$

$$(2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

$$(ii) (B \cap C) = \{3, 4\} \cap \{4, 5, 6\} = \{4\}$$

$$\therefore A \times (B \cap C) = \{1, 2, 3\} \times \{4\} = \{(1, 4), (2, 4), (3, 4)\}$$

$$(iii) (A \times B) = \{1, 2, 3\} \times \{3, 4\}$$

$$= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

$$(B \times C) = \{3, 4\} \times \{4, 5, 6\}$$

$$= \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$$

$$\therefore (A \times B) \cap (B \times C) = \{(3, 4)\}$$

18. For non empty sets A and B prove that $(A \times B) = (B \times A) \Leftrightarrow A = B$

Ans. First we assume that $A = B$

Then $(A \times B) = (A \times A)$ and $(B \times A) = (A \times A)$

$$\therefore (A \times B) = (B \times A)$$

This, when $A = B$, then $(A \times B) = (B \times A)$

Conversely, Let $(A \times B) = (B \times A)$, and let be $x \in A$.

Then, $x \in A \Rightarrow (x, b) \in A \times B$ for some $b \in B$

$$\Rightarrow (x, b) \in B \times A \quad [\because A \times B = B \times A]$$

$$\Rightarrow x \in B.$$

$$\therefore A \subseteq B$$

similarly, $B \subseteq A$

Hence, $A = B$

19. Let m be a given fixed positive integer. let

$R = \{(a, b) : a, b \in \mathbb{Z} \text{ and } (a - b) \text{ is divisible by } m\}$ show that R is an equivalence relation on \mathbb{Z} .

Ans. $R = \{(a, b) : a, b \in \mathbb{Z} \text{ and } (a - b) \text{ is divisible by } m\}$

(i) Let $a \in \mathbb{Z}$. Then,

$a - a = 0$, which is divisible by m

$\therefore (a, a) \in R$ for all $a \in \mathbb{Z}$

so R is reflexive

(ii) Let $(a, b) \in R$ Then

$(a, b) \in R \Rightarrow (a - b)$ is divisible by m

$\Rightarrow -(a - b)$ is divisible by m

$\Rightarrow (b - a)$ is divisible by m

$$\Rightarrow (b, a) \in R$$

$$\text{Then } (a, b) \in R \Rightarrow (b, a) \in R.$$

So R is symmetric.

(iii) Let $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow (a - b) \text{ is divisible by } m \text{ and } (b - c) \text{ is divisible by } m$$

$$\Rightarrow [(a - b) + (b - c)] \text{ is divisible by } m$$

$$\Rightarrow (a - c) \text{ is divisible by } m$$

$$\Rightarrow (a, c) \in R$$

$$\therefore (a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R.$$

So, R is transitive this R is reflexive symmetric and transitive Hence, R is an equivalence relation and Z .

20. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 4\}$ let R be the relation, is greater than from A to B . Write R as a set of ordered pairs. find domain (R) and range (R)

$$\text{Ans. } R = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)\}$$

$$\text{Domain of } R = \{2, 3, 4, 5\} \text{ Range of } R = \{1, 2, 3, 4\}$$

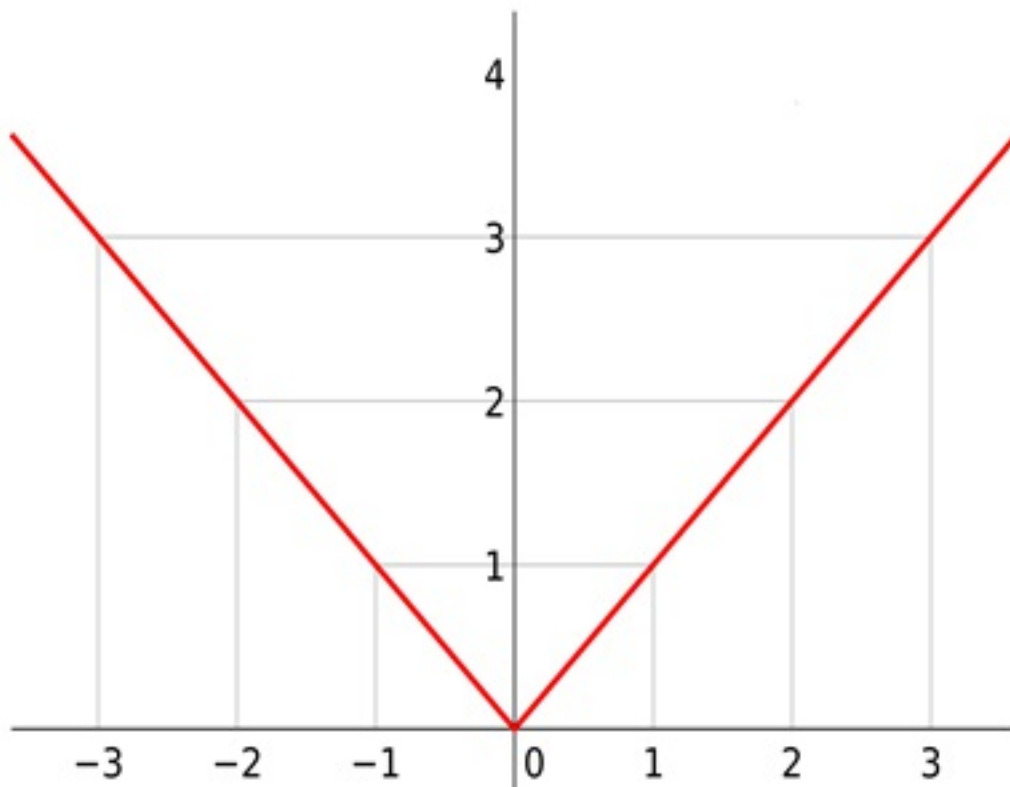
21. Define modulus function Draw graph.

$$\text{Ans. let } f: R \rightarrow R: f(x) = |x| \text{ for each } x \in R. \text{ then } f(x) = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

we know that $|x| > 0$ for all x

$\therefore \text{dom}(f) = \mathbb{R}$ and $\text{range}(f) = \text{set of non negative real number}$

Drawing the graph of modulus function defined by



$$f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = |x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

We have

x	3	-2	-1	0	1	2	3	4
$f(x)$	3	2	1	0	1	2	3	4

Scale: 5 small divisions = 1 unit

On a graph paper, we plot the points

$A(-3,3), B(-2,2), C(-1,1), o(0,0), D(1,1), E(2,2), F(3,3)$ and $G(4,4)$

Join them successively to obtain the graph lines AO and OG, as show in the figure above.

22. Let $f(x) = \begin{cases} x^2, & \text{when } 0 \leq x \leq 3 \\ 3x, & \text{when } 3 \leq x \leq 10 \end{cases}$ $g(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 2x, & 3 \leq x \leq 10 \end{cases}$ Show that f is a

function, while g is not a function.

Ans. Each element in $\{0, 10\}$ has a unique image under f .

But, $g(3) = 3^2 = 9$ and

$$g(3) = (2 \times 3) = 6$$

So g is not a function

23. Let $A = \{1, 2\}$ and $B = \{3, 4\}$ write $A \times B$ how many subsets will $A \times B$ have? List them.

Ans. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$; 16 Subsets of $A \times B$ have

$$\text{Subsets} = \phi, \{(1, 3)\}, \{(1, 4)\}, \{(2, 3)\}, \{(2, 4)\},$$

$$\{(1, 4)\}, \{(1, 3), (2, 3)\}, \{(1, 3), (2, 4)\},$$

$$\{(1, 4), (2, 3)\}, \{(1, 4), (2, 4)\}, \{(2, 3)\},$$

$$\{(2, 4)\}, \{(1, 3), (1, 4), (2, 3)\}, \{(1, 3), (1, 4)\},$$

$$\{(2, 4)\}, \{(1, 3), (2, 3), (2, 4)\}, \{(1, 4), (2, 3)\},$$

$$\{(2, 4)\};$$

24. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (ii) $A \times C$ is subset of $B \times D$

Ans. L.H.S. $B \cap C = \phi$

Part-I

$$\text{L.H.S } A \times (B \cap C) = \phi$$

$$\text{R.H.S. } A \times B = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4) \\ (2,1), (2,2), (2,3), (2,4) \end{array} \right\}$$

$$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$$

$$(A \times B) \cap (A \times C) = \phi \quad \text{L.H.S} = \text{R.H.S}$$

Part-II

$$B \times D = \left\{ \begin{array}{l} (1,5), (1,6), (1,7), (1,8) \\ (2,5), (2,6), (2,7), (2,8) \end{array} \right\}$$

$$(A \times C) \subset (B \times D)$$

25. Find the domain and the range of the relation R defined by

$$R = [(x+1, x+3) : x \in (0, 1, 2, 3, 4, 5)]$$

Ans. Given $x \in \{0, 1, 2, 3, 4, 5\}$

$$\text{put } x=0, x+1=0+1=1 \text{ and } x+3=0+3=3$$

$$x=1, x+1=1+1=2 \text{ and } x+3=1+3=4,$$

$$x=2, x+1=2+1=3 \text{ and } x+3=2+3=5,$$

$$x=3, x+1=3+1=4 \text{ and } x+3=3+3=6,$$

$$x=4, x+1=4+1=5 \text{ and } x+3=4+3=7$$

$$x=5, x+1=6 \text{ and } x+3=5+3=8$$

$$\text{Hence } R = [(1,3), (2,4), (3,5), (4,6), (5,7), (6,8)]$$

$$\therefore \text{Domain of } R = [1, 2, 3, 4, 5, 6] \text{ and range of } R = [3, 4, 5, 6, 7, 8]$$

26. Find the linear relation between the components of the ordered pairs of the relation R where $R = \{(2,1), (4,7), (1,-2), \dots\}$

Ans. Given $R = \{(2,1), (4,7), (1,-2), \dots\}$

Let $y = ax + b$ be the linear relation between the components of R

Since $(2,1) \in R, \therefore y = ax + b \Rightarrow 1 = 2a + b \dots \dots (i)$

Also $(4,7) \in R, \therefore y = ax + b \Rightarrow 7 = 4a + b \dots \dots (ii)$

Subtracting (i) from (ii), we get $2a = 6 \Rightarrow a = 3$

Subtracting $a = 3$ in (i), we get $1 = 6 + b \Rightarrow b = -5$

Substituting these values of a and b in $y = ax + b$, we get

$y = 3x - 5$, which is the required linear relation between the components of the given relation.

27. Let $A = \{1, 2, 3, 4, 5, 6\}$ define a relation R from A to A by

$$R = \{(x, y) : y = x + 1, x, y \in A\}$$

(i) write R in the roster form

(ii) write down the domain, co-domain and range of R

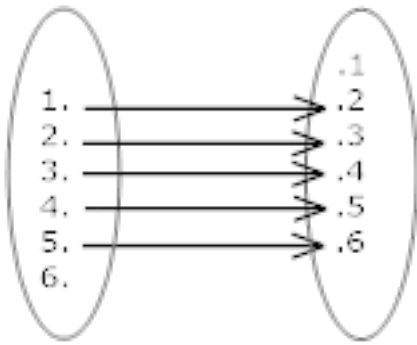
(iii) Represent R by an arrow diagram

Ans. (i) $\{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

(ii) Domain = $\{1, 2, 3, 4, 5\}$ co domain = A , range = $\{2, 3, 4, 5, 6\}$

(iii)





28. A relation ' f ' is defined by $f : x \rightarrow x^2 - 2$ where $x \in \{-1, -2, 0, 2\}$

(i) list the elements of f

(ii) is f a function?

Ans. Relation f is defined by $f : x \rightarrow x^2 - 2$

(i) is $f(x) = x^2 - 2$ where $x \in \{-1, -2, 0, 2\}$

$$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$$

$$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$$

$$f(0) = 0^2 - 2 = 0 - 2 = -2$$

$$f(2) = 2^2 - 2 = 4 - 2 = 2$$

$$\therefore f = \{(-1, -1), (-2, 2), (0, -2), (2, 2)\}$$

(ii) We note that each element of the domain of f has a unique image; therefore, the relation f is a function.

29. If $y = \frac{6x-5}{5x-6}$. Prove that $f(y) = x$, $x \neq \frac{6}{5}$

Ans. $y = \frac{6x-5}{5x-6}$

$$y = f(x) = \frac{6x-5}{5x-6}$$

$$f(y) = \frac{6 \left[\frac{6x-5}{5x-6} \right] - 5}{5 \left[\frac{6x-5}{5x-6} \right] - 6}$$

$$f(y) = \frac{36x - \cancel{30} - 25x + \cancel{30}}{\cancel{5x} - 6} = \frac{30x - 25 - \cancel{30x} + 36}{\cancel{5x} - 6}$$

$$f(y) = \frac{11x}{11} = x, x \neq \frac{6}{5}$$

30. Let $f : X \rightarrow Y$ be defined by $f(x) = x^2$ for all $x \in X$ where $X = \{-2, -1, 0, 1, 2, 3\}$ and $Y = \{0, 1, 4, 7, 9, 10\}$ write the relation f in the roster form. Is f a function?

Ans. $f : X \rightarrow Y$ defined by

$$f(x) = x^2, x \in X$$

$$\text{and } X = \{-2, -1, 0, 1, 2, 3\}$$

$$Y = \{0, 1, 4, 7, 9, 10\}$$

$$f(-2) = (-2)^2 = 4$$

$$f(-1) = (-1)^2 = 1$$

$$f(0) = 0^2 = 0$$

$$f(1) = 1^2 = 1$$

$$f(2) = 2^2 = 4$$

$$f(3) = 3^2 = 9$$

$$\therefore f = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$$

f is a function because different elements of X have different images in Y

31. Determine a quadratic function 'f' defined by

$$f(x) = ax^2 + bx + c \text{ if } f(0) = 6, f(2) = 11 \text{ and } f(-3) = 6$$

Ans. $f(x) = ax^2 + bx + c$

$$f(0) = 6$$

$$a \times 0^2 + b \times 0 + c = 6$$

$$c = 6$$

$$f(2) = 11$$

$$a \times 2^2 + b \times 2 + c = 11$$

$$4a + 2b + c = 11$$

$$4a + 2b + 6 = 11$$

$$4a + 2b = 11 - 6$$

$$[4a + 2b = 5] \text{ --- (i)}$$

$$f(-3) = 6$$

$$a \times (-3)^2 + b \times (-3) + c = 6$$

$$9a - 3b + 6 = 6$$

$$[9a - 3b = 0] \text{ --- (ii)}$$

Multiplying eq. (i) by 3 and eq. (ii) by 2

$$12a + 6b = 15$$

$$18a - 6b = -12$$

$$\hline 30a = 3$$

$$a = \frac{3}{30} = \frac{1}{10}$$

$$4 \times \frac{1}{5} + 2b = 5$$

$$2b = 5 - \frac{2}{5}$$

$$2b = \frac{25-2}{5} = \frac{23}{5}$$

$$b = \frac{23}{10}$$

$$\therefore f(x) = \frac{1}{10}x^2 + \frac{23}{10}x + 6$$

32. Find the domain and the range of the function f defined by $f(x) = \frac{x+2}{|x+2|}$

Ans. $f(x) = \frac{x+2}{|x+2|}$

For Df, $f(x)$ must be a real no.

$$\Rightarrow |x+2| \neq 0 \Rightarrow x+2 \neq 0 \Rightarrow x \neq -2$$

\therefore Domain of f = set of all real numbers

except -2 i.e. $Df = \mathbb{R} - \{-2\}$

for Rf

case I if $x+2 > 0$ then $|x+2| = x+2$

$$\therefore f(x) = \frac{x+2}{|x+2|} = 1$$

case II if $x+2 < 0$, $|x+2| = -(x+2)$



$$\therefore f(x) = \left(\frac{\cancel{x+2}}{\cancel{-x+2}} \right) = -1$$

\therefore Range of $f = \{-1, 1\}$

33. Find the domain and the range of $f(x) = \frac{x^2}{1+x^2}$

Ans. $f(x) = \frac{x^2}{1+x^2}$

Domain of $f =$ all real no. $= R$

for Range let $f(x) = y$

$$y = \frac{x^2}{1+x^2}$$

$$y(1+x^2) = x^2$$

$$y + yx^2 = x^2$$

$$y = x^2 - yx^2$$

$$y = (1-y)x^2$$

$$x^2 = \frac{y}{1-y}$$

$$x = \sqrt{\frac{y}{1-y}}$$

$$\frac{y}{1-y} \geq 0 \quad 1-y \neq 0$$

$$y \neq 1$$

also $y \geq 0$ and $1-y > 0$

$$y < 1$$

\therefore Range of $f = [0, 1)$.

34. If

$$A = \{1, 2, 3\}, B = \{1, 2, 3, 4\} \text{ and}$$

$$R = \{(x, y) : (x, y) \in A \times B, y = x + 1\} \text{ then}$$

(i) find $A \times B$ (ii) write domain and Range

Ans.

(i)

$$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)\}$$

$$(ii) R = \{(1, 2), (2, 3), (3, 4)\}$$

$$\text{Domain of } R = \{1, 2, 3\}$$

$$\text{Range of } R = \{2, 3, 4\}$$



CBSE Class 12 Mathematics
Important Questions
Chapter 2
Relations and Functions

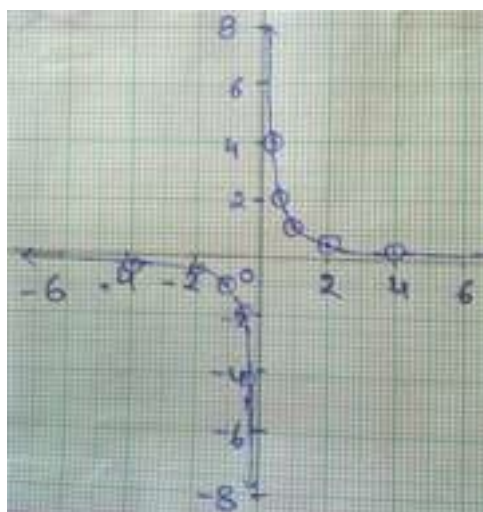
6 Marks Questions

1. Draw the graphs of the following real functions and hence find their range

$$f(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$$

Ans. Given $f(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$

Let $y = f(x) = \frac{1}{x}$ or $y = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$



(Fig for Answer 11)

x	-4	-2	-1	-0.5	-0.25	0.5	1	2	4
$y = \frac{1}{x}$	-0.25	-0.5	-1	-2	-4	2	1	0.5	0.25

Plot the points shown in the above table and join these points by a free hand drawing.



Portion of the graph are shown the right margin

From the graph, it is clear that $Rf = R - [0]$

This function is called reciprocal function.

2.If $f(x) = x - \frac{1}{x}$, Prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

Ans. If $f(x) = x - \frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + f\left(\frac{1}{x}\right)$

Given $f(x) = x - \frac{1}{x}$, $Df = R - [0]$

$$\Rightarrow f(x^3) = x^3 - \frac{1}{x^3} \text{ and } f\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{\frac{1}{x}} = \frac{1}{x} - x \dots\dots (i)$$

$$\therefore [f(x)]^3 = \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} + 3\left(\frac{1}{x} - x\right)$$

$$= f(x^3) + 3f\left(\frac{1}{x}\right) [\text{using (i)}]$$

3.Draw the graphs of the following real functions and hence find their range

$$(i) f(x) = 2x - 1 \quad (ii) f(x) = \frac{x^2 - 1}{x - 1}$$

Ans. (i) Given $f(x)$ i.e. $y = x - 1$, which is first degree equation in x, y and hence it represents a straight line. Two points are sufficient to determine straight line uniquely

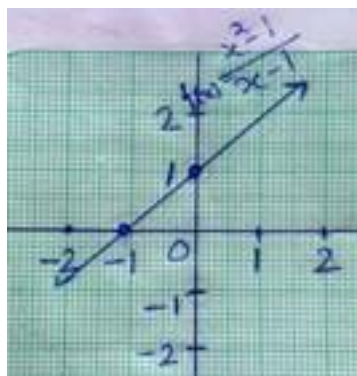
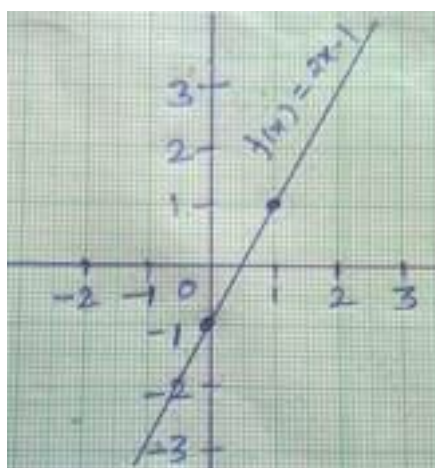


Table of values

x	0	1
y	-1	1

A portion of the graph is shown in the figure from the graph, it is clear that y takes all real values. It therefore that $R_F = R$

(ii) Given $f(x) = \frac{x^2 - 1}{x - 1} \Rightarrow D_F = R - (1)$



Let $y = f(x) = \frac{x^2 - 1}{x - 1} = x + 1 (\because x \neq 1)$

i.e. $y = x + 1$, which is a first degree equation in x, y and hence it represents a straight line. Two points are sufficient to determine a straight line uniquely

Table of values

x	-1	0
y	0	1

A portion of the graph is shown in the figure from the graph it is clear that y takes all real values except 2. It follows that $R_f = R - [2]$.

4. Let f be a function defined by $f : x \rightarrow 5x^2 + 2, x \in R$

(i) find the image of 3 under f

(ii) find $f(3) + f(2)$

(iii) find x such that $f(x) = 22$

Ans. Given $f(x) = 5x^2 + 2, x \in R$

(i) $f(3) = 5 \times 3^2 + 2 = 5 \times 9 + 2 = 47$

(ii) $f(2) = 5 \times 2^2 + 2 = 5 \times 4 + 2 = 22$

$\therefore f(3) \times f(2) = 47 \times 22 = 1034$

(iii) $f(x) = 22$

$\Rightarrow 5x^2 + 2 = 22$

$\Rightarrow 5x^2 = 20$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2, -2$

5. The function $f(x) = \frac{9x}{5} + 32$ is the formula to connect $x^\circ C$ to Fahrenheit



units find (i) $f(0)$ (ii) $f(-10)$ (iii) the value of x $f(x) = 212$ interpret the result in each case

Ans. $f(x) = \frac{9x}{5} + 32$ (given)

(i) $f(0) = \left(\frac{9 \times 0}{5} + 32 \right) = 32 \Rightarrow f(0) = 32 \Rightarrow 0^\circ C = 32^\circ F$

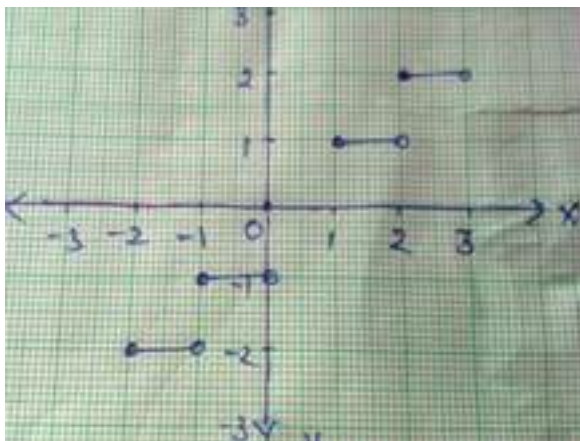
(ii) $f(-10) = \left(\frac{9 \times (-10)}{5} + 32 \right) = 14 \Rightarrow f(-10) = 14^\circ \Rightarrow (-10)^\circ C = 14^\circ F$

(iii) $f(x) = 212 \Leftrightarrow \frac{9x}{5} + 32 = 212 \Leftrightarrow 9x = 5 \times (180)$
 $\Leftrightarrow x = 100$

$\therefore 212^\circ F = 100^\circ C$

6. Draw the graph of the greatest integer function, $f(x) = [x]$.

Ans. Clearly, we have



$$f(x) = \left\{ \begin{array}{l} -2, \text{ when } x \in [-2, -1) \\ -1, \text{ when } x \in [-1, 0) \\ 0, \text{ when } x \in [0, 1) \\ 1, \text{ when } x \in [1, 2) \end{array} \right\}$$

x	$-2 \leq x < 1$	$-1 \leq x < 0$	$0 \leq x < 1$	$1 \leq x < 2$	$2 \leq x < 3$
y	-2	-1	0	1	2

7. Find the domain and the range of the following functions:

(i) $f(x) = \sqrt{x^2 - 4}$ (ii) $f(x) = \sqrt{16 - x^2}$ (iii) $f(x) = \frac{1}{\sqrt{9 - x^2}}$

Ans. (i) Given $f(x) = \sqrt{x^2 - 4}$

For D_f , $f(x)$ must be a real number

$\Rightarrow \sqrt{x^2 - 4}$ Must be a real number

$\Rightarrow x^2 - 4 \geq 0 \Rightarrow (x + 2)(x - 2) \geq 0$

\Rightarrow either $x \leq -2$ or $x \geq 2$

$\Rightarrow D_f = (-\infty, -2] \cup [2, \infty)$.

For R_f , let $y = \sqrt{x^2 - 4}$(i)

As square root of a real number is always non-negative, $y \geq 0$

On squaring (i), we get $y^2 = x^2 - 4$

$\Rightarrow x^2 = y^2 + 4$ but $x^2 \geq 0$ for all $x \in D_f$

$\Rightarrow y^2 + 4 \geq 0 \Rightarrow y^2 \geq -4$, which is true for all $y \in \mathbb{R}$. also $y \geq 0$

$\Rightarrow R_f = [0, \infty)$

(ii) Given $f(x) = \sqrt{16 - x^2}$

For D_f , $f(x)$ must be a real number

$\Rightarrow \sqrt{16-x^2}$ must be a real number

$\Rightarrow \sqrt{16-x^2} \geq 0 \Rightarrow -(x^2-16) \geq 0$

$\Rightarrow x^2-16 \leq 0$

$\Rightarrow (x+4)(x-4) \leq 0 \Rightarrow -4 \leq x \leq 4$

$\Rightarrow D_f = [-4, 4]$.

For R_f , let $y = \sqrt{16-x^2}$ (i)

As square root of real number is always non-negative, $y \geq 0$

Squaring (i) we get

$$y^2 = 16 - x^2$$

$\Rightarrow x^2 = 16 - y^2$ but $x^2 \geq 0$ for all $x \in D_f$

$\Rightarrow 16 - y^2 \geq 0 \Rightarrow -(y^2 - 16) \geq 0 \Rightarrow y^2 - 16 \leq 0$

$\Rightarrow (y+4)(y-4) \leq 0 \Rightarrow -4 \leq y \leq 4$ but $y \geq 0$

$\Rightarrow R_f = [0, 4]$

(iii) Given $f(x) = \frac{1}{\sqrt{9-x^2}}$

For D_f , $f(x)$ must be a real number

$\Rightarrow \frac{1}{\sqrt{9-x^2}}$ must be a real number

$$\Rightarrow 9 - x^2 > 0 \Rightarrow -(x^2 - 9) > 0 \Rightarrow x^2 - 9 < 0$$

$$\Rightarrow (x+3)(x-3) < 0 \Rightarrow -3 < x < 3 \Rightarrow D_f = (-3, 3)$$

For R_f , let $y = \frac{1}{\sqrt{9-x^2}}, y \neq 0 \dots \dots (i)$

Also as the square root of a real number is always non-negative, $y > 0$.

on squaring (i) we get

$$y^2 = \frac{1}{9-x^2} \Rightarrow 9-x^2 = \frac{1}{y^2} \Rightarrow x^2 = 9 - \frac{1}{y^2}$$

But $x^2 \geq 0$ for all $x \in D_f \Rightarrow 9 - \frac{1}{y^2} \geq 0$

$$y^2 > 0$$

(Multiply both sides by y^2 , a positive real number)

$$\Rightarrow 9y^2 - 1 \geq 0 \Rightarrow y^2 - \frac{1}{9} \geq 0$$

$$\Rightarrow \left(y + \frac{1}{3}\right)\left(y - \frac{1}{3}\right) \geq 0$$

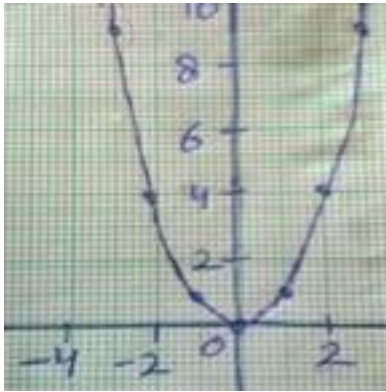
$$\Rightarrow \text{either } y \leq -\frac{1}{3} \text{ or } y \geq \frac{1}{3}$$

$$y > 0 \Rightarrow y \geq \frac{1}{3}$$

$$\Rightarrow R_f = \left[\frac{1}{3}, \infty\right).$$

8. Draw the graphs of the following real functions and hence find range: $f(x) = x^2$

Ans.



Given $f(x) = x^2 \Rightarrow D_f = R$

Let $y = f(x) = x^2, x \in R$

x	-4	-3	-2	-1	0	1	2	3	4
$y = x^2$	16	9	4	1	0	1	4	9	16

Plot the points

$(-4, 16), (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16), \dots$

And join these points by a free hand drawing. A portion of the graph is shown in sigma (next)

From the graph, it is clear that y takes all non-negative real values, it follows that

$$R_f = [0, \infty)$$

9. Define polynomial function. Draw the graph of $f(x) = x^3$ find domain and range

Ans. A function $f: R \rightarrow R$ define by

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where $a_0, a_1, a_2, \dots, a_n \in R$

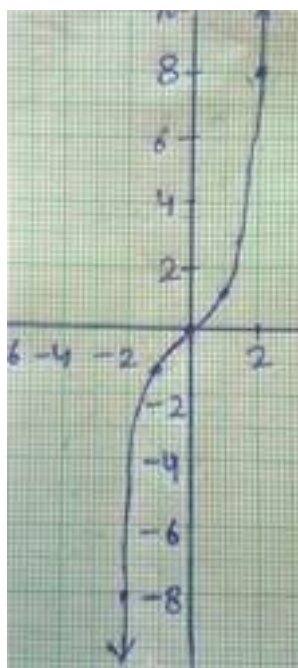
And n is non negative integer is called polynomial function

Graph of $f(x) = x^3$

x	0	1	2	-1	-2
$f(x)$	0	1	8	-1	-8

Domain of $f = \mathbb{R}$

Range of $f = \mathbb{R}$



10.(a) If A, B are two sets such that $n(A \times B) = 6$ and some elements of $A \times B$ are $(-1, 2), (2, 3), (4, 3)$, then find $A \times B$ and $B \times A$

(b) Find domain of the function $f(x) = \frac{1}{\sqrt{x + [x]}}$

Ans. (a) Given A and B are two sets such that

$$n(A \times B) = 6$$

Some elements of $A \times B$ are

$$(-1, 2), (2, 3) \text{ and } (4, 3)$$

$$\text{then } A = \{-1, 2, 4\} \text{ and } B = \{2, 3\}$$

$$A \times B = \{(-1, 2), (-1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$$

$$B \times A = \{(2, -1), (3, -1), (2, 2), (3, 2), (2, 4), (3, 4)\}$$

(b)

$$f(x) = \frac{1}{\sqrt{x + [x]}}$$

we knowe that

$$x + [x] > 0 \text{ for all } x > 0$$

$$x + [x] = 0 \text{ for all } x = 0$$

$$x + [x] < 0 \text{ for all } x < 0$$

also $f(x) = \frac{1}{\sqrt{x + [x]}}$ is defined for all

$$x \text{ satisfying } x + [x] > 0$$

$$\text{Hence, Domain } (f) = (0, \infty)$$